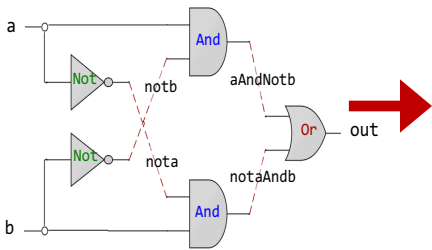
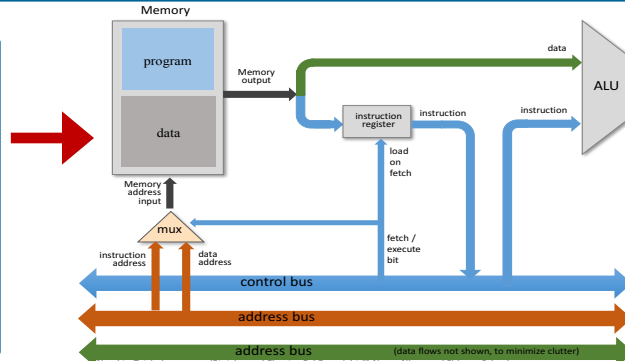




Digital Logic Design



```
CHIP Xor {
  IN a, b;
  OUT out;
  PARTS:
    Not(in=a, out=nota);
    Not(in=b, out=notb);
    And(a=nota, b=b, out=w1);
    And(a=a, b=notb, out=w2);
    Or(a=w1, b=w2, out=out);
}
```



@R1
D=M
@temp
M=D

```
0000000000000001
1111110000010000
0000000000010000
1110001100001000
```

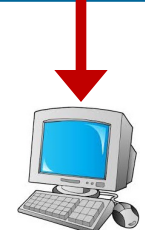
Lecture # 17, 18

Design of Synchronous Sequential Circuits

```
#include<stdio.h>
#include<stdlib.h>
int main(){
  printf("Learning is fun with Arif\n");
  exit(0);
}
```

```
global main
SECTION .data
  msg: db "Learning is fun with Arif", 0Ah, 0h
  len_msg: equ $ - msg
SECTION .text
main:
  mov rax,1
  mov rdi,1
  mov rsi,msg
  mov rdx,len_msg
  syscall
  mov rax,60
  mov rdi,0
  syscall
```

```
0: b8 01 00 00 00
5: bf 01 00 00 00
a: 48 be 00 00 00 00 00
11: 00 00 00
14: ba 1b 00 00 00
19: 0f 05
1b: b8 3c 00 00 00
20: bf 00 00 00 00
25: 0f 05
```



Slides of first half of the course are adapted from:
<https://www.nand2tetris.org>
 Download s/w tools required for first half of the course from the following link:
<https://drive.google.com/file/d/0B9c0BdDz6XpZUh3X2dPR1o0MUE/view>

Instructor: Muhammad Arif Butt, Ph.D.



Today's Agenda



Steps of Designing Synchronous Sequential Circuits

1. Construct **State Diagram** from problem statement
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



Example 1

Problem Statement: Design a sequence recognizer circuit that detects an input sequence of '1011'. The sequence recognizer outputs a '1' on the detection of this input sequence



Step 1: Construct State Diagram

- **Problem Statement:** Design a sequence recognizer circuit that detects an input sequence of '1011'. The sequence recognizer outputs a '1' on the detection of this input sequence

| | | | | | | | | | | | | | | | | | | | | |
|---------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Input | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| Output | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

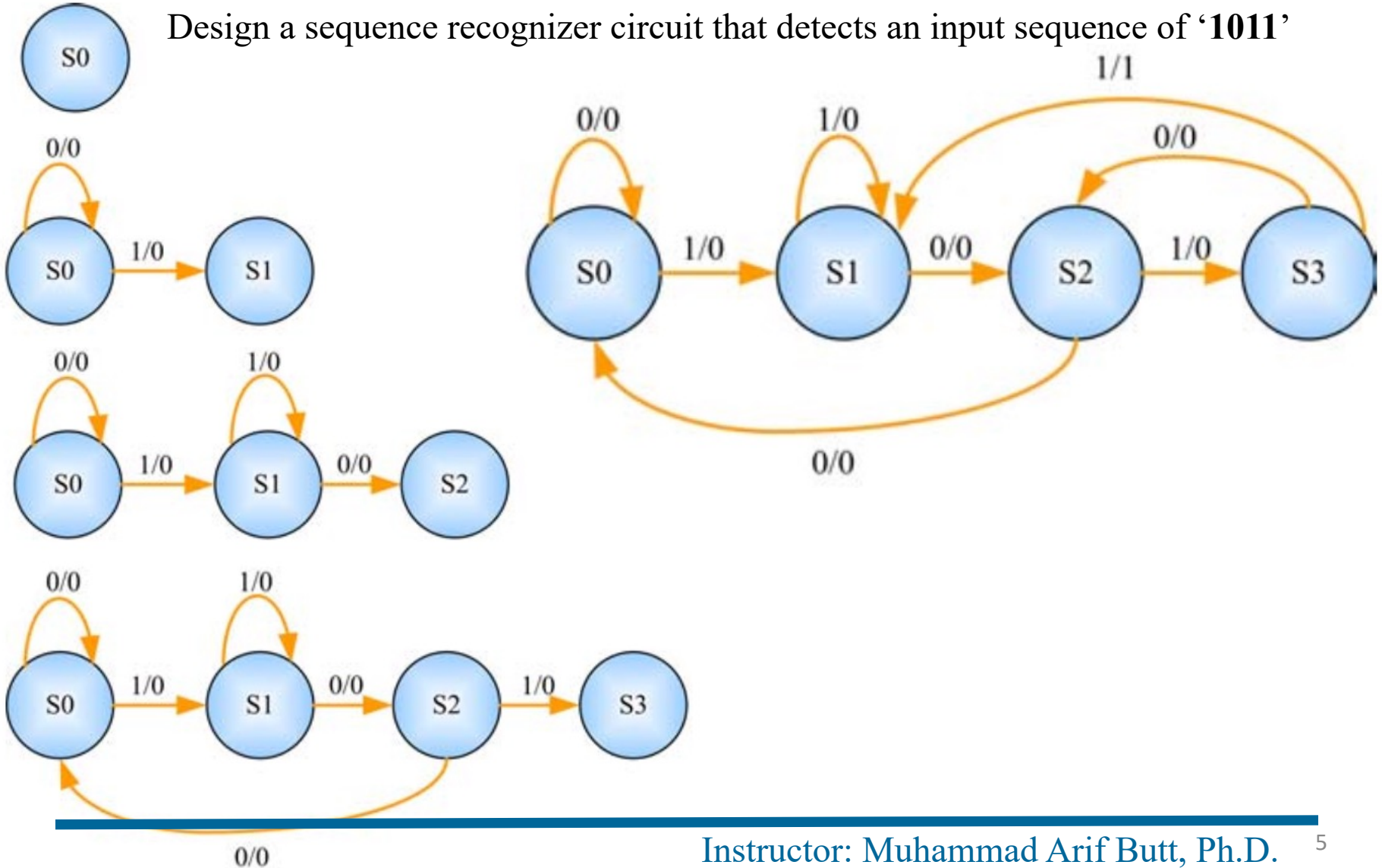
- **State Diagram:** A state diagram consists of circles (which represent the states) and directed arcs that connect the circles and represent the transitions between states. In a state diagram:
 - The number of circles is equal to the number of states. Every state is given a label (or a binary encoding) written inside the corresponding circle
 - The number of arcs leaving any circle is 2^n , where n is the number of inputs of the sequential circuit
 - The label of each arc has the notation x/y , where x is the input vector that causes the state transition, and y is the value of the output during that present state
 - An arc may leave a state and end up in the same or any other state



Step 1: Construct State Diagram (cont...)

| | | | | | | | | | | | | | | | | | | | | |
|---------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Input | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| Output | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

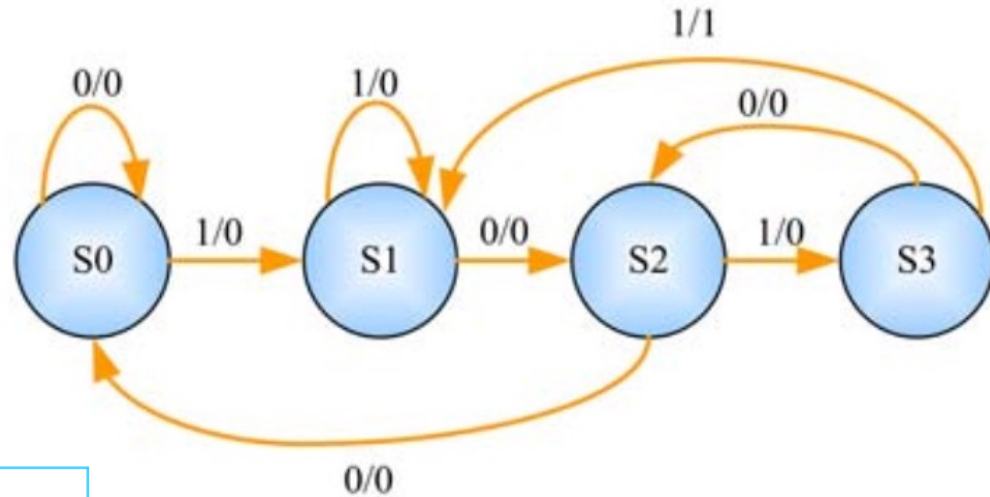
Design a sequence recognizer circuit that detects an input sequence of '1011'





Step 2: Derive State Table

- A **State Table** represents the time sequence of inputs, outputs and states in a tabular form. The state table of the given state diagram is given below:



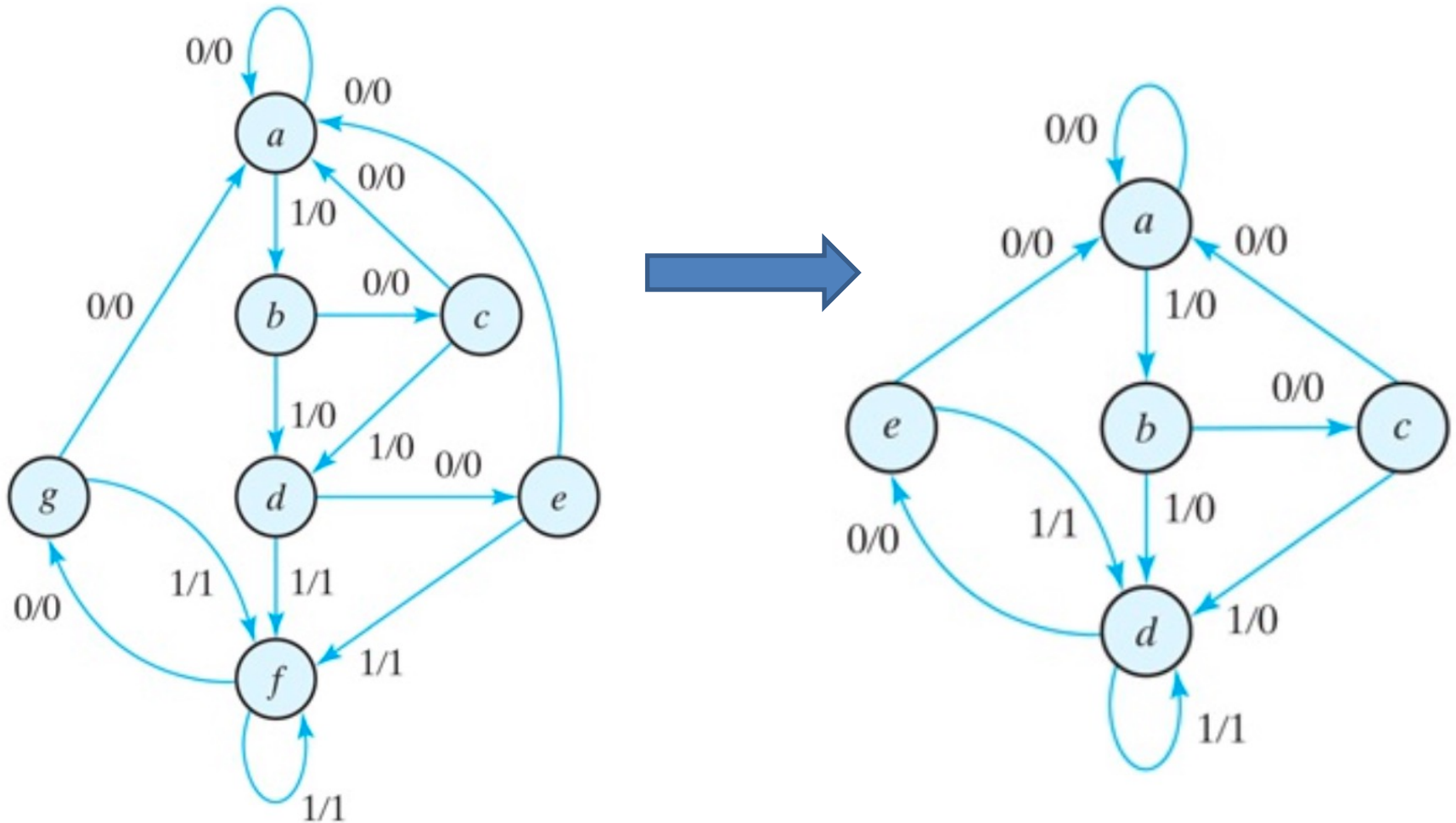
| Inputs of Combinational Circuit | | Next State | Output |
|---------------------------------|-------|------------|--------|
| Present State | Input | | |
| <i>S0</i> | 0 | <i>S0</i> | 0 |
| <i>S0</i> | 1 | <i>S1</i> | 0 |
| <i>S1</i> | 0 | <i>S2</i> | 0 |
| <i>S1</i> | 1 | <i>S1</i> | 0 |
| <i>S2</i> | 0 | <i>S0</i> | 0 |
| <i>S2</i> | 1 | <i>S3</i> | 0 |
| <i>S3</i> | 0 | <i>S2</i> | 0 |
| <i>S3</i> | 1 | <i>S1</i> | 1 |

| Present State | Next State | | Output | |
|---------------|------------|-----------|--------|-----|
| | X=0 | X=1 | X=0 | X=1 |
| <i>S0</i> | <i>S0</i> | <i>S1</i> | 0 | 0 |
| <i>S1</i> | <i>S2</i> | <i>S1</i> | 0 | 0 |
| <i>S2</i> | <i>S0</i> | <i>S3</i> | 0 | 0 |
| <i>S3</i> | <i>S2</i> | <i>S1</i> | 0 | 1 |



Step 3: State Reduction

- State reduction means reducing the states of the state diagram, while keeping the input and output requirements unchanged





Step 4: State Assignment and Number of FF

State assignment is the process of assigning binary codes to states. In the state diagram since we have four states, so two bit code is used. If there are unused states, then they are treated as don't care conditions during design.

| <i>State</i> | <i>Assignment</i> |
|--------------|-------------------|
| <i>S0</i> | 00 |
| <i>S1</i> | 01 |
| <i>S2</i> | 10 |
| <i>S3</i> | 11 |

State Table with Symbolic States

| Inputs of Combinational Circuit | | Next State | Output |
|---------------------------------|-------|------------|--------|
| Present State | Input | | |
| <i>S0</i> | 0 | <i>S0</i> | 0 |
| <i>S0</i> | 1 | <i>S1</i> | 0 |
| <i>S1</i> | 0 | <i>S2</i> | 0 |
| <i>S1</i> | 1 | <i>S1</i> | 0 |
| <i>S2</i> | 0 | <i>S0</i> | 0 |
| <i>S2</i> | 1 | <i>S3</i> | 0 |
| <i>S3</i> | 0 | <i>S2</i> | 0 |
| <i>S3</i> | 1 | <i>S1</i> | 1 |

State Table with Binary States

| Inputs of Combinational Circuit | | Next State | Output |
|---------------------------------|----------|------------|----------|
| Present State | Input | | |
| <i>A B</i> | <i>X</i> | <i>A B</i> | <i>Y</i> |
| 0 0 | 0 | 0 0 | 0 |
| 0 0 | 1 | 0 1 | 0 |
| 0 1 | 0 | 1 0 | 0 |
| 0 1 | 1 | 0 1 | 0 |
| 1 0 | 0 | 0 0 | 0 |
| 1 0 | 1 | 1 1 | 0 |
| 1 1 | 0 | 1 0 | 0 |
| 1 1 | 1 | 0 1 | 1 |



Step 5: Construct Excitation Table

- Suppose we decide to use JK and D Flip Flops.

| Q_t | Q_{t+1} | J | K |
|-------|-----------|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | X |
| 1 | 0 | X | 1 |
| 1 | 1 | X | 0 |

| Q_t | Q_{t+1} | D |
|-------|-----------|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| <i>Present State</i> | | <i>Input</i> | <i>Next State</i> | | <i>o/p</i> | <i>Flip-flops Inputs</i> | | |
|----------------------|----------|--------------|-------------------|----------|------------|--------------------------|----------------------|----------------------|
| <i>A</i> | <i>B</i> | | <i>A</i> | <i>B</i> | | <i>J_A</i> | <i>K_A</i> | <i>D_B</i> |
| 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | |
| 0 | 0 | 1 | 0 | 1 | 0 | X | 1 | |
| 0 | 1 | 0 | 1 | 0 | 1 | X | 0 | |
| 0 | 1 | 1 | 0 | 1 | 0 | X | 1 | |
| 1 | 0 | 0 | 0 | 0 | X | 1 | 0 | |
| 1 | 0 | 1 | 1 | 1 | 0 | X | 1 | |
| 1 | 1 | 0 | 1 | 0 | 0 | X | 0 | |
| 1 | 1 | 1 | 0 | 1 | 1 | X | 1 | |



Step 6: Derive Circuit o/p and FF i/p Equations

| | | | | |
|----------|-----------|----|----|----|
| | BX | | | |
| A | 00 | 01 | 11 | 10 |
| 0 | | | | 1 |
| 1 | X | X | X | X |

$$J_A = BX'$$

| | | | | |
|----------|-----------|----|----|----|
| | BX | | | |
| A | 00 | 01 | 11 | 10 |
| 0 | X | X | X | X |
| 1 | 1 | | 1 | |

$$K_A = BX + B'X'$$

| | | | | |
|----------|-----------|----|----|----|
| | BX | | | |
| A | 00 | 01 | 11 | 10 |
| 0 | | 1 | 1 | |
| 1 | | 1 | 1 | |

$$D_B = X$$

| | | | | |
|----------|-----------|----|----|----|
| | BX | | | |
| A | 00 | 01 | 11 | 10 |
| 0 | | | | |
| 1 | | | | 1 |

$$Y = ABX$$

| <i>Present State</i> | <i>Input</i> | <i>Next State</i> | <i>o/p</i> | <i>Flip-flops Inputs</i> | | |
|----------------------|--------------|-------------------|------------|--------------------------|----------|----------|
| | | | | <i>A</i> | <i>B</i> | <i>Y</i> |
| 0 0 | 0 | 0 0 | 0 | 0 | X | 0 |
| 0 0 | 1 | 0 1 | 0 | 0 | X | 1 |
| 0 1 | 0 | 1 0 | 0 | 1 | X | 0 |
| 0 1 | 1 | 0 1 | 0 | 0 | X | 1 |
| 1 0 | 0 | 0 0 | 0 | X | 1 | 0 |
| 1 0 | 1 | 1 1 | 0 | X | 0 | 1 |
| 1 1 | 0 | 1 0 | 0 | X | 0 | 0 |
| 1 1 | 1 | 0 1 | 1 | X | 1 | 1 |



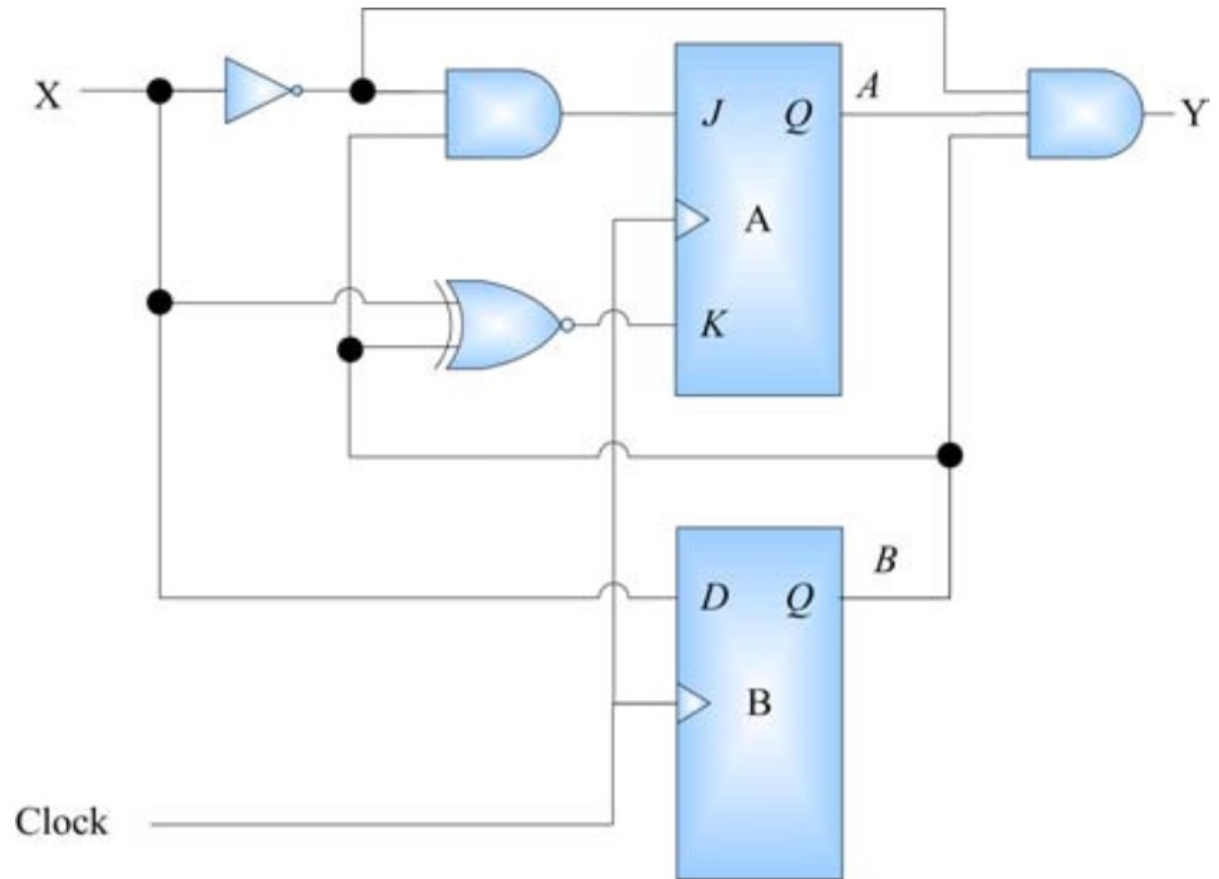
Step 7: Circuit Diagram

$$JA = BX'$$

$$KA = BX + B'X'$$

$$DB = X$$

$$Y = ABX$$

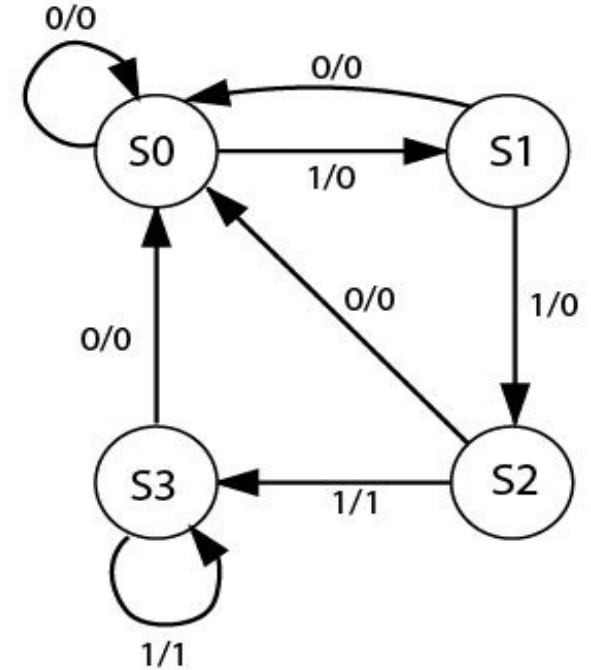




Example 2

Problem Statement: Design a sequence recognizer circuit that detects three or more consecutive 1s in a string of bits coming through an input line. The output becomes 1 on detection of every third consecutive 1, otherwise it remains 0.

- Use D Flip Flops
- Use JK Flip Flops
- Use T Flip Flops



Steps of Designing Synchronous Sequential Circuits

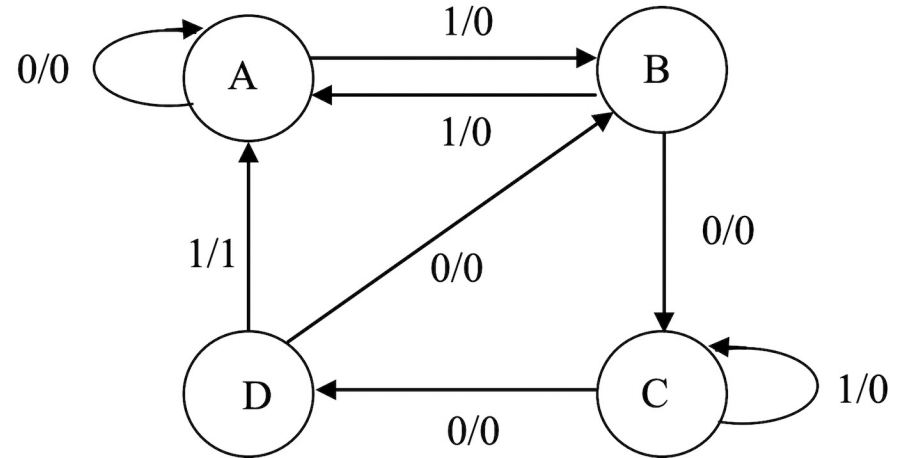
1. Construct **State Diagram** from problem statement
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



Example 3

Problem Statement: Design the sequential circuit of following state diagram using:

- Use D Flip Flops
- Use JK Flip Flops
- Use T Flip Flops



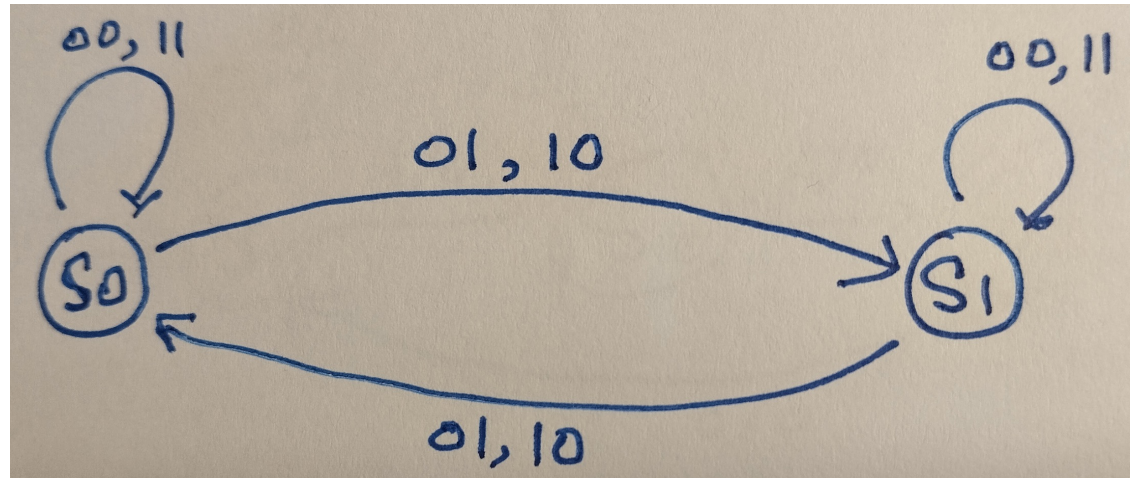
Steps of Designing Synchronous Sequential Circuits

1. Construct **State Diagram** from problem statement
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



Example 4

- Design a synchronous sequential circuit having two inputs and no output. It does not change the state on an input of 00 and 11. A transition occurs when the input vector is 01 or 10.
- Use D Flip Flop
- Use JK Flip Flop
- Use T Flip Flop



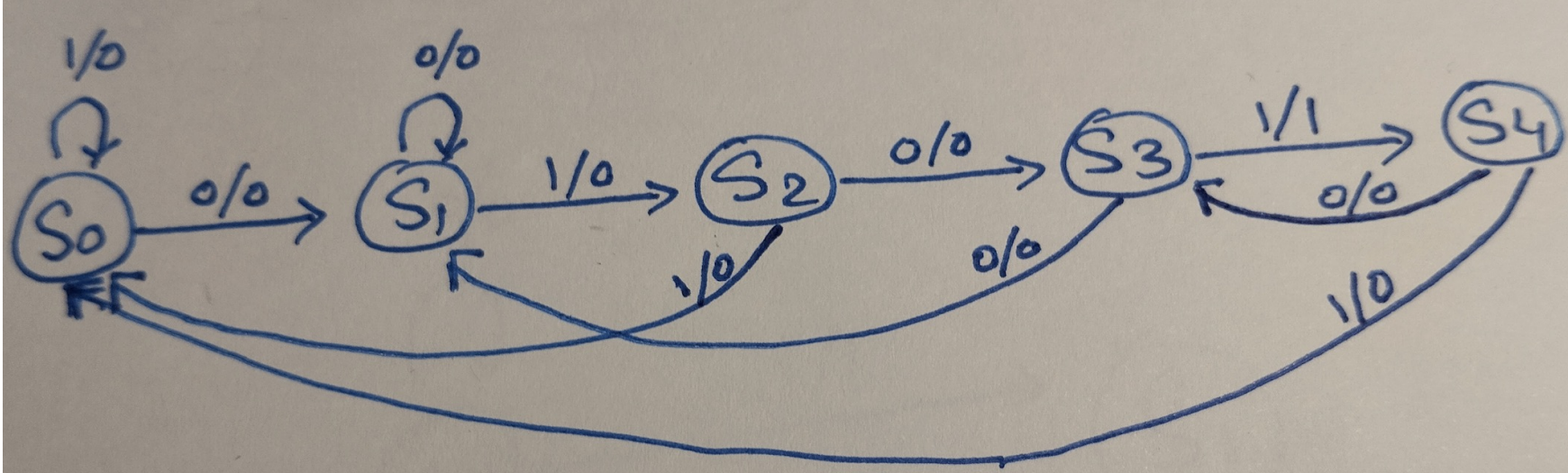
Steps

1. Construct **State Diagram**
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



Example 5

- Design a synchronous sequential circuit having a single input and single output. The output is equal to zero unless an input sequence **0101** is received.



Steps of Designing Synchronous Sequential Circuits

1. Construct **State Diagram** from problem statement
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



Counters

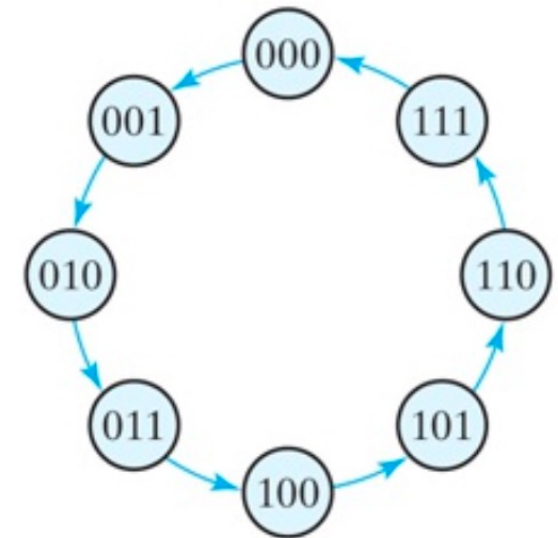
- A counter is a special type of register that goes through a pre-determined sequence of states upon the application of input pulses
- **Counters are used for:**
 - Counting the number of occurrences of an event
 - Keeping time or calculating amount of time between events
 - Baud rate generation
- **A w-bit counter consists of two main elements:**
 - A w-bit register to store a w-bit value
 - A combinational logic to
 - Compute the next value (according to a specific counting function)
 - Load a new value of user/programmer choice
 - Reset the counter to a default value
- **Categories of Counters:**
 - **Asynchronous/Ripple Counter:** The output of each Flip Flop provides the clock signal for the next Flip Flop
 - **Synchronous Counter:** All the Flip Flops changes their state simultaneously and are capable of operating at higher frequencies
- **Examples:** Up/Down Binary Counters, BCD Counter(s), Gray Code Counter, Ring Counter, Johnson Counter



Example 6 (Counters)

Problem Statement: Design a 3 bit binary up counter, which changes its state from 000 to 111 automatically and then restarts. No input is required for transition. Moreover, there is no output. Use

- D Flip Flops
- T Flip Flops
- JK Flip Flops



Steps of Designing Synchronous Sequential Circuits

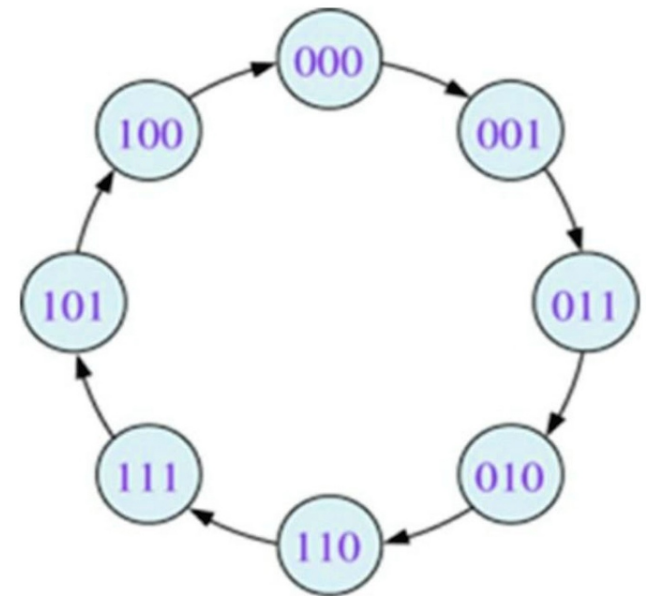
1. Construct **State Diagram** from problem statement
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



Example 7 (Counters)

Problem Statement: Design a 3 bit Grey Code counter, which changes its state from 000 to 100 automatically and then restarts. No input is required for transition. Moreover, there is no output. Use

- D Flip Flops
- T Flip Flops
- JK Flip Flops



Steps of Designing Synchronous Sequential Circuits

1. Construct **State Diagram** from problem statement
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



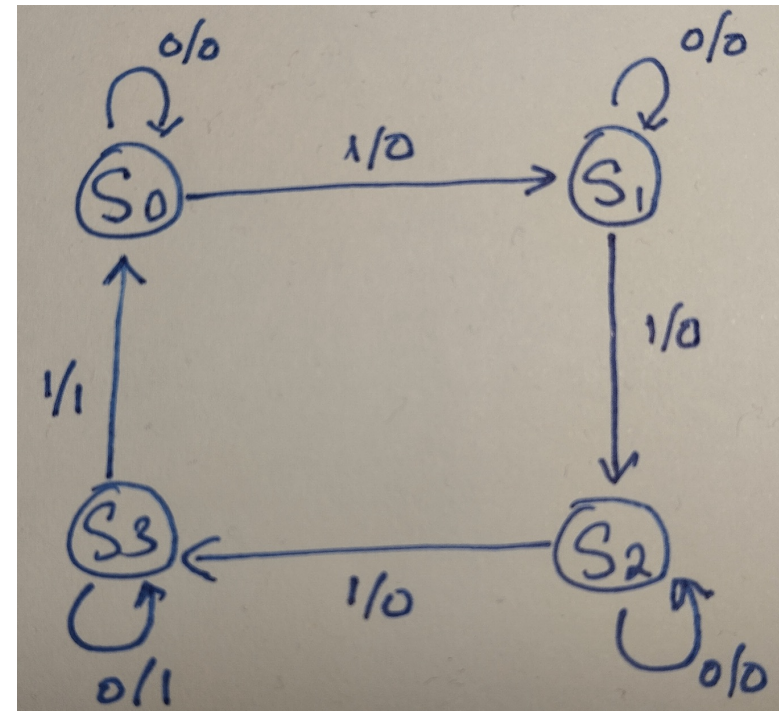
Example 8 (Counters)

Design a synchronous sequential circuit having a single input and single output. As long as input is 1, the circuit behaves as a binary counter with a sequence of states 00, 01, 10, 11 and back to 00. Output is equal to 1 when the present state is 11. Use

- D Flip Flops
- T Flip Flops
- JK Flip Flops

Steps

1. Construct **State Diagram**
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. **State Assignment** and **number** of FF
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**





Example 9 (Counters)

Problem Statement: Design a 3 bit sequence counter, which changes its states as 0-5-3-7-6-2-0 (in decimal). Any other state not in sequence should take the counter to state 0. No input is required for transition. Moreover, there is no output. Use

- D Flip Flops
- T Flip Flops
- JK Flip Flops

Steps of Designing Synchronous Sequential Circuits

1. Construct **State Diagram** from problem statement
2. Derive **State Table** from state diagram
3. Perform **State Reduction**, if possible (optional)
4. Do **State Assignment** and decide on **number** of Flip Flops to be used
5. Construct **Excitation Table**
6. Derive **Circuit output and Flip Flop input equations**
7. Draw the **Circuit Diagram**



Mealy vs Moore Machines

- The output of a **Mealy Machine** is determined by both its current state and current inputs
- The output of a **Moore machine** is determined by its current state only

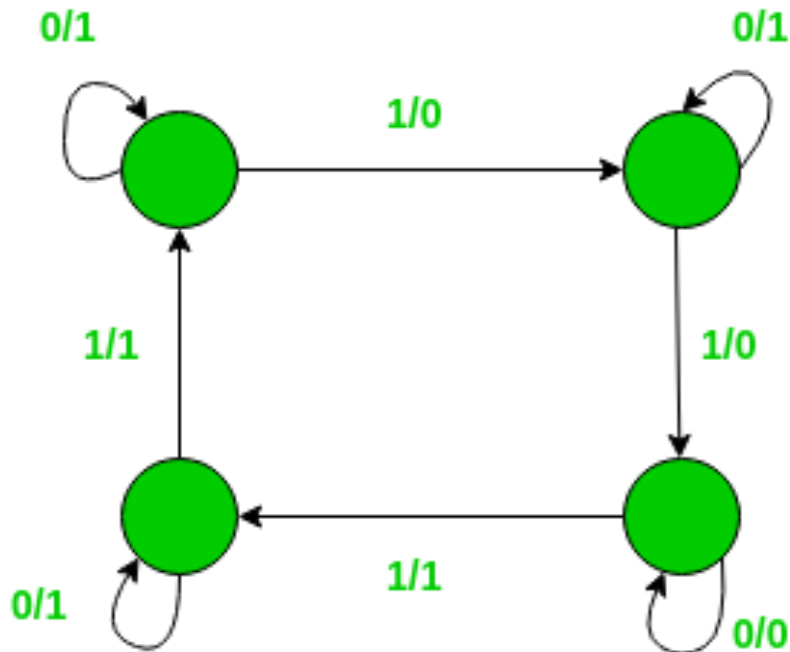


Figure - Mealy machine

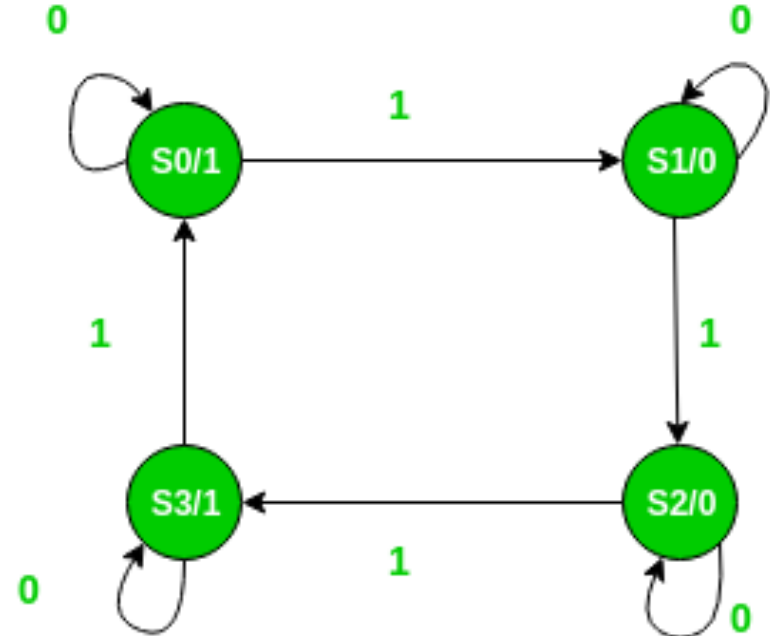


Figure - Moore machine



Things To Do

- Practice
- Practice
- Practice
- Practice
- Practice
- Practice
- Practice
- Practice



Coming to office hours does NOT mean you are academically weak!